Solving Systems using Laplace Transforms

Lesson #19

6CT.5-7

Homework

- Problems: 3.3,

  A LTI system is described as

  $$H(s) = 5.263\frac{s+1.9}{(s+10)(s+1)}$$

  Find the impulse response : $h(t)$

  Draw the Bode plot for $0.01 \le f \le 100$cycles/min

- 3.4b,

  The unit impulse response is given as

  $$h(t) = (0.7e^{-5t} + 0.2e^{-t} + 0.1e^{-0.1t})u(t)$$

  Find the response due to a unit step function.

Homework

- Problems: 3.12c

  A LTI system is described as

  $\dot{y} + 3y = x(t)$

  Find the response due to $(1 + 2e^{-t})u(t)$

- 3.13c

  A LTI system is described as

  $\ddot{y} + 8\dot{y} + 15y = 5x(t)$

  Find the response due to a unit step : $x(t) = u(t)$

Homework

- Problems: 3.18a&c

  A LTI system is described as

  $$Y(s) = \frac{2}{s(s^2 + 3s + 2)}$$

  Find the poles of the system

  Find $y(t)$

- 3.23

  A LTI system is described as

  $$H(s) = \frac{10}{(s+4)(s+2)^3}$$

  Find $h(t)$

Homework

- Find the inverse transforms for:

$$(a)\,\frac{1-e^{-3s}}{s(s+2)};(b)\,\frac{-s^2+9s+10}{s(s+2)(s+5)}$$

$$(c)\,\frac{90s-800}{(s^2+100)(s+4)};(d)\,\frac{s+2}{[(s+1)^2+1]^2}$$

$$(e)\,\frac{s^3+8s^2+18s+12}{(s^2+2s+2)(s^2+6s+10)};(f)\,\frac{s^2+4s+3}{(s+2)(s+4)}$$

Homework Answers #1

- Problems: 3.3

$$H(s)=\frac{K(s+1.9)}{(s+10)(s+1)}=\frac{5.263(s+1.9)}{(s+10)(s+1)}=\frac{K_1}{s+10}+\frac{K_2}{s+1}$$

$$K_1=\frac{5.263(s+1.9)}{(s+1)}\Big|_{s=-10}=\frac{5.263(-10+1.9)}{(-10+1)}=4.737$$

$$K_2=\frac{5.263(s+1.9)}{(s+10)}\Big|_{s=-1}=\frac{5.263(-1+1.9)}{(-1+10)}=.5263$$

$$H(s)=\frac{4.737}{s+10}+\frac{.5263}{s+1}$$

$$h(t)=[4.737e^{-10t}+.5263e^{-t}]u(t)$$

$$H(j\omega)=\frac{5.263(j\omega+1.9)}{(j\omega+10)(j\omega+1)}=\frac{5.263(\sqrt{\omega^2+1.9^2})}{(\sqrt{\omega^2+10^2})(\sqrt{\omega^2+1^2})}\angle\tan^{-1}(\frac{\omega}{1.9})-[\tan^{-1}(\frac{\omega}{10})+\tan^{-1}(\frac{\omega}{1})]$$

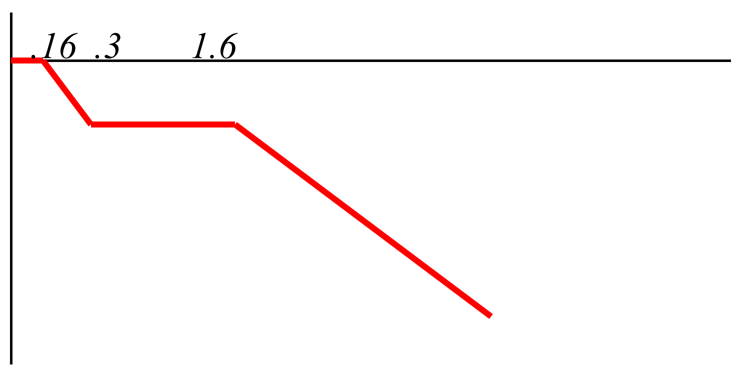
$$f_z=\frac{1.9}{2\pi}=0.3Hz$$

$$f_{p1}=\frac{1}{2\pi}=0.16Hz$$

$$f_{p2}=\frac{10}{2\pi}=1.6Hz$$

$$H(j0)=.99997\angle 0^\circ$$

$$H(j\omega)_{\omega\to\infty}=0\angle -90^\circ$$



Homework Answers #2

- Problems: 3.4b

$$h(t) = (0.7e^{-5t} + 0.2e^{-t} + 0.1e^{-0.1t})u(t)$$

$$H(s) = \frac{(s+.32)(s+2.07)}{(s+5)(s+1)(s+.1)s} = \frac{K_1}{s} + \frac{K_2}{s+5} + \frac{K_3}{s+1} + \frac{K_4}{s+.1}$$

$$K_1 = \frac{(s+.32)(s+2.07)}{(s+5)(s+1)(s+.1)}\Big|_{s=0} = \frac{(.32)(2.07)}{(5)(1)(.1)} = 1.325$$

$$K_2 = \frac{(s+.32)(s+2.07)}{(s+1)(s+.1)s}\Big|_{s=-5} = \frac{(-5+.32)(-5+2.07)}{(-5+1)(-5+.1)(-5)} = -.14$$

$$K_3 = \frac{(s+.32)(s+2.07)}{(s+5)(s+.1)s}\Big|_{s=-1} = \frac{(-1+.32)(-1+2.07)}{(-1+5)(-1+.1)(-1)} = -0.202$$

$$K_4 = \frac{(s+.32)(s+2.07)}{(s+5)(s+1)s}\Big|_{s=-.1} = \frac{(-.1+.32)(-.1+2.07)}{(-.1+5)(-.1+1)(-.1)} = -.98$$

$$x(t) = [1.325 - .14e^{-5t} - 0.202e^{-t} + -.98e^{-.1t}]u(t)$$

Homework Answers #3

- Problems: 3.12c

$$c)\,\pounds[\dot{y}(t)+3y(t)]=\pounds[x(t)]$$

$$x(t)=(1+2e^{-t})u(t)$$

$$X(s)=\frac{1}{s}+\frac{2}{s+1}=\frac{3s+1}{s(s+1)}$$

$$(s+3)Y(s)=\frac{3s+1}{s(s+1)}$$

$$Y(s)=\frac{3s+1}{s(s+1)(s+3)}=\frac{K_1}{(s+3)}+\frac{K_2}{(s+1)}+\frac{K_3}{s}$$

$$K_1=\frac{3s+1}{s(s+1)}\Big|_{s=-3}=\frac{-9+1}{(-3)(-2)}=-\frac{8}{6}=-\frac{4}{3}$$

$$K_2=\frac{3s+1}{s(s+3)}\Big|_{s=-1}=\frac{-3+1}{-1(2)}=1$$

$$K_3=\frac{3s+1}{(s+1)(s+3)}\Big|_{s=0}=\frac{1}{3}$$

$$Y(s)=\frac{-4/3}{(s+3)}+\frac{1}{(s+1)}+\frac{1/3}{s}$$

$$y(t)=(-\frac{4}{3}e^{-3t}+e^{-t}+\frac{1}{3})u(t)$$

Homework Answers #4

- Problems: 3.13b

$$c)\pounds[\ddot{y}(t)+8\dot{y}(t)+15y(t)]=\pounds[5x(t)]$$

$$(s^2+8s+15)Y(s)=5X(s)$$

$$Y(s)=\frac{5X(s)}{(s^2+8s+15)}=\frac{5}{s(s+3)(s+5)}=\frac{K_1}{s}+\frac{K_2}{s+3}+\frac{K_3}{s+5}$$

$$K_1=\frac{5}{(s+3)(s+5)}\Big|_{s=0}=\frac{5}{(3)(5)}=\frac{1}{3}$$

$$K_2=\frac{5}{s(s+5)}\Big|_{s=-3}=\frac{5}{(-3)(2)}=-\frac{5}{6}$$

$$K_3=\frac{5}{s(s+3)}\Big|_{s=-5}=\frac{5}{(-5)(-2)}=\frac{1}{2}$$

$$Y(s)=\frac{1}{3}\frac{1}{s}-\frac{5}{6}\frac{1}{s+3}+\frac{1}{2}\frac{1}{s+5}$$

$$y(t)=(\frac{1}{3}-\frac{5}{6}e^{-3t}+\frac{1}{2}e^{-5t})u(t)$$

Homework Answers #5

- Problems: 3.18a&c

$$a)Y(s)=\frac{2}{s(s^2+3s+2)}$$

$$Y(s)=\frac{2}{s(s+1)(s+2)}$$

$$b)Y(s)=\frac{2}{s(s+1)(s+2)}=\frac{K_1}{s}+\frac{K_2}{s+1}+\frac{K_3}{s+2}$$

$$K_1=\frac{2}{(s+1)(s+2)}\Big|_{s=0}=\frac{2}{(1)(2)}=1$$

$$K_2=\frac{2}{(s)(s+2)}\Big|_{s=-1}=\frac{2}{(-1)(1)}=-2$$

$$K_3=\frac{2}{(s)(s+1)}\Big|_{s=-2}=\frac{2}{(-2)(-1)}=1$$

$$Y(s)=\frac{1}{s}-\frac{1}{s+1}+\frac{1}{s+2}$$

$$y(t)=(1-2e^{-t}+e^{-2t})u(t)$$

Homework Answers #6

- Problems: 3.23

$$H(s) = \frac{10}{(s+4)(s+2)^3} = \frac{K_1}{s+4} + \frac{K_2}{(s+2)^3} + \frac{K_3}{(s+2)^2} + \frac{K_4}{(s+2)}$$

$$K_1 = \frac{10}{(s+2)^3}\Big|_{s=-4} = \frac{10}{(-2)^3} = -\frac{10}{8} = -\frac{5}{4}$$

$$K_2 = \frac{10}{(s+4)}\Big|_{s=-2} = \frac{10}{(2)} = 5$$

$$K_3 = \frac{d}{ds}\frac{10}{(s+4)}\Big|_{s=-2} = -\frac{10}{(s+4)^2}\Big|_{s=-2} = -\frac{10}{4} = -\frac{5}{2}$$

$$K_3 = \frac{1}{2}\frac{d^2}{ds^2}\frac{10}{(s+4)}\Big|_{s=-2} = \frac{1}{2}\frac{d}{ds} - \frac{10}{(s+4)^2}\Big|_{s=-2} = \frac{1}{2}\frac{20}{(s+4)^3}\Big|_{s=-2} = \frac{10}{8} = \frac{5}{4}$$

$$H(s) = -\frac{5/4}{s+4} + \frac{5}{(s+2)^3} - \frac{5/2}{(s+2)^2} + \frac{5/4}{(s+2)}$$

$$h(t) = \{-\frac{5}{4}e^{-4t} + (\frac{5}{2}t^2 - \frac{5}{2}t + \frac{5}{4})e^{-2t}\}u(t)$$

Homework Answers #7

- Find the inverse transforms for:

$$(a)\ £^{-1}[\frac{1-e^{-3s}}{s(s+2)}] = £^{-1}[\frac{1}{s(s+2)}] - £^{-1}[\frac{e^{-3s}}{s(s+2)}]$$

$$£^{-1}[\frac{1}{s(s+2)}] = £^{-1}[\frac{K_1}{s} + \frac{K_2}{s+2}]$$

$$K_1 = \frac{1}{(s+2)}|_{s=0} = \frac{1}{2}$$

$$K_2 = \frac{1}{s}|_{s=-2} = -\frac{1}{2}$$

$$£^{-1}[\frac{1}{s(s+2)}] = \frac{1}{2}£^{-1}[\frac{1}{s} - \frac{1}{s+2}] = \frac{1}{2}[1-e^{-2t}]u(t)$$

$$£^{-1}[\frac{e^{-3s}}{s(s+2)}] = \frac{1}{2}[1-e^{-2(t-3)}]u(t-3)$$

$$£^{-1}[\frac{1-e^{-3s}}{s(s+2)}] = \frac{1}{2}\{[1-e^{-2t}]u(t) - [1-e^{-2(t-3)}]u(t-3)\}$$

Homework Answers #8

- Find the inverse transforms for:

$$(b)£^{-1}[\frac{-s^2+9s+10}{s(s+2)(s+5)}] = £^{-1}[\frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+5}]$$

$$K_1 = \frac{-s^2+9s+10}{(s+2)(s+5)}\Big|_{s=0} = \frac{10}{(2)(5)} = 1$$

$$K_2 = \frac{-s^2+9s+10}{s(s+5)}\Big|_{s=-2} = \frac{-4-18+10}{(-2)(3)} = \frac{-12}{-6} = 2$$

$$K_3 = \frac{-s^2+9s+10}{s(s+2)}\Big|_{s=-5} = \frac{-25-45+10}{-5(-3)} = \frac{-60}{15} = -4$$

$$£^{-1}[\frac{-s^2+9s+10}{s(s+2)(s+5)}] = [1+2(e^{-2t} - 2e^{-5t})]u(t)$$

Homework Answers #9

- Find the inverse transforms for:

$$(c)\mathcal{£}^{-1}\left[\frac{90s-800}{(s^2+100)(s+4)}\right]=\mathcal{£}^{-1}\left[\frac{\mathbf{K}_1}{s\text{-}j10}+\frac{\mathbf{K}_1{}^*}{s+j10}+\frac{K_2}{s+4}\right]$$

$$\mathbf{K}_1=\frac{90s-800}{(s+j10)(s+4)}\Big|_{s=j10}=\frac{j900-800}{(j20)(j10+4)}=\frac{j45-40}{(-10+j4)}$$

$$=\frac{-40+j45}{-10+j4}=\frac{60.2\angle-48.4^\circ}{10.8\angle-21.8^\circ}=5.6\angle-26.6^\circ$$

$$\mathbf{K}_1{}^*=5.6\angle 26.6^\circ$$

$$K_2=\frac{90s-800}{(s^2+100)}\Big|_{s=-4}=\frac{-360-800}{(16+100)}=\frac{-1160}{116}=-10$$

$$\mathcal{£}^{-1}\left[\frac{90s-800}{(s^2+100)(s+4)}\right]=[11.2\cos(10t-26.6^\circ)-10e^{-4t}]u(t)$$

Homework Answers #10

- Find the inverse transforms for:

$$(d)\ £^{-1}[\frac{s+2}{[(s+1)^2+1]^2}] = £^{-1}\{\frac{K_1}{[(s+1)^2+1]^2} + \frac{K_2}{[(s+1)^2+1]}\}$$

$$= £^{-1}\{\frac{\mathbf{K_1}}{[(s+1-j)]^2} + \frac{\mathbf{K_1}^*}{[(s+1+j)]^2} + \frac{\mathbf{K}_2}{[(s+1-j)]} + \frac{\mathbf{K}_2^*}{[(s+1+j)]}\}$$

$$\mathbf{K_1} = \frac{s+2}{[(s+1+j)]^2}\Big|_{s=-1+j} = \frac{-1+j+2}{[(-1+j+1+j)]^2} = \frac{1+j}{(2j)^2} = \frac{1}{4}(-1-j) = \frac{\sqrt{2}}{4}\angle 225^\circ$$

$$\mathbf{K_1}^* = \frac{1}{4}(-1-j) = \frac{\sqrt{2}}{4}\angle 135^\circ$$

$$\mathbf{K}_2 = \frac{d}{ds}\frac{s+2}{[(s+1+j)]^2}\Big|_{s=-1+j} = \{\frac{1}{[(s+1+j)]^2} - \frac{2(s+2)}{[(s+1+j)]^3}\}\Big|_{s=-1+j} = \frac{1}{[(2j)]^2} - \frac{2(-1+j+2)}{[(2j)]^3}$$

$$= \frac{1}{-4} - \frac{2(-1+j+2)}{-8j} = -\frac{1}{4}(1 - \frac{1+j}{j}) = -\frac{1}{4}(1+j-1) = -\frac{1}{4}j = \frac{1}{4}\angle -90^\circ$$

$$\mathbf{K}_2^* = \frac{1}{4}\angle 90^\circ$$

$$£^{-1}[\frac{s+2}{[(s+1)^2+1]^2}] = £^{-1}\{\frac{\frac{\sqrt{2}}{4}\angle 225^\circ}{[(s+1+j)]^2} + \frac{\frac{\sqrt{2}}{4}\angle 135^\circ}{[(s+1-j)]^2} + \frac{\frac{1}{4}\angle -90^\circ}{[(s+1+j)]} + \frac{\frac{1}{4}\angle 90^\circ}{[(s+1-j)]}\}$$

$$= [\frac{1}{2}e^{-t}\{\sqrt{2}t\cos(t+225^\circ) + \cos(t-90^\circ)\}]u(t)$$

Homework Answers #11

- Find the inverse transforms for:

$$(e)\ \pounds^{-1}[\frac{s^3+8s^2+18s+12}{(s^2+2s+2)(s^2+6s+10)}] = \pounds^{-1}[\frac{s^3+8s^2+18s+12}{[(s+1)^2+1][(s+3)^2+1]}] =$$

$$\pounds^{-1}[\frac{s^3+8s^2+18s+12}{[(s+1+j)(s+1-j)][(s+3+j)(s+3-j)]}] = \pounds^{-1}[\frac{\mathbf{K}_1}{s+1+j}+\frac{\mathbf{K}_1{}^*}{s+1-j}+\frac{\mathbf{K}_2}{s+3+j}+\frac{\mathbf{K}_2{}^*}{s+3-j}]$$

$$\mathbf{K}_1 = \frac{s^3+8s^2+18s+12}{[(s+1-j)][(s+3+j)(s+3-j)]}\Big|_{s=-1-j} = \frac{(-1-j)^3+8(-1-j)^2+18(-1-j)+12}{[-2j)][(2)(2-2j)]}$$

$$\text{Note: } (-1-j) = \sqrt{2}e^{j\frac{5\pi}{4}}$$

$$(-1-j)^2 = (\sqrt{2}\angle -135^\circ)^2 = 2e^{j\frac{5\pi}{2}} = 2j$$

$$(-1-j)^3 = 2\sqrt{2}\angle -405^\circ = 2\sqrt{2}e^{j\frac{15\pi}{4}} = 2\sqrt{2}e^{j\frac{7\pi}{4}} = 2-2j$$

$$\mathbf{K}_1 = \frac{2-2j+16(j)+18(-1-j)+12}{[-2j)][(2)(2-2j)]} = \frac{-4-4j}{-8-8j} = \frac{1}{2}$$

$$\mathbf{K}_2 = \frac{s^3+8s^2+18s+12}{[(s+1+j)][(s+1-j)(s+3-j)]}\Big|_{s=-3-j} = \frac{(-3-j)^3+8(-3-j)^2+18(-3-j)+12}{[-2)][(-2-2j)(-2j)]}$$

$$\text{Note: } (-3-j) = \sqrt{10}e^{j\tan^{-1}(\frac{-1}{-3})} = \sqrt{10}e^{-j2.82}$$

$$(-3-j)^2 = 10e^{-j5.64} = 8+6j$$

$$(-3-j)^3 = 10\sqrt{10}e^{-j8.46} = -18-26j$$

$$\mathbf{K}_2 = \frac{-18-26j+8(8+6j)+18(-3-j)+12}{[-2)][(-2-2j)(-2j)]} = \frac{4+4j}{8-j8} = \frac{4\sqrt{2}\angle 45^\circ}{8\sqrt{2}\angle -45^\circ} = .5\angle 90^\circ$$

$$\pounds^{-1}[\frac{\mathbf{K}_1}{s+1+j}+\frac{\mathbf{K}_1{}^*}{s+1-j}+\frac{\mathbf{K}_2}{s+3+j}+\frac{\mathbf{K}_2{}^*}{s+3-j}] = [0.5e^{-t}\cos(t)+e^{-3t}\sin(t)]u(t)$$

Homework Answers #12

- Find the inverse transforms for:

$$(f)\ \pounds^{-1}[\frac{s^2+4s+3}{(s+2)(s+4)}] = 1 + \frac{K_1}{s+2} + \frac{K_2}{s+4}$$

$$K_1 = \frac{s^2+4s+3}{(s+4)}\Big|_{s=-2} = \frac{4-8+3}{2} = -\frac{1}{2}$$

$$K_2 = \frac{s^2+4s+3}{(s+2)}\Big|_{s=-4} = \frac{16-16+3}{-2} = -\frac{3}{2}$$

$$\pounds^{-1}[\frac{s^2+4s+3}{(s+2)(s+4)}] = \pounds^{-1}[1 + \frac{1}{2}\{\frac{1}{s+2} - \frac{3}{s+4}\}]$$

$$= \delta(t) + \frac{1}{2}\{e^{-2t} - 3e^{-4t}\}u(t)$$